Published by Institute of Physics Publishing for SISSA/ISAS
Received: *December 26, 2003*
Accepted: *January 16, 2004*

D-term strings

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Abstract: We study the embedding of cosmic strings, related to the Abrikosov-Nielsen-Olesen vortex solution, into $d = 4$, $N = 1$ supergravity. We find that the local cosmic string solution which saturates the BPS bound of supergravity with D-term potential for the Higgs field and with constant Fayet–Iliopoulos term, has 1/2 of supersymmetry unbroken. We observe an interesting relation between the gravitino supersymmetry transformation, positive energy condition and the deficit angle of the cosmic string. We argue that the string solutions with magnetic flux with F-term potential cannot be supersymmetric, which leads us to a conjecture that D_1-strings (wrapped D_{1+q}-branes) of string theory in the effective $4d$ supergravity are described by the D-term strings that we study in this paper. We give various consistency checks of this conjecture, and show that it highlights some generic properties of non-BPS string theory backgrounds, such as brane-anti-brane systems. Supersymmetry breaking by such systems can be viewed as FI D-term breaking, which implies, under certain conditions, the presence of gauged R-symmetry on such backgrounds. The D-term nature of the brane-anti-brane energy can also provide information on the superpotential for the tachyon, which Higgses the R-symmetry. In this picture, the inter-brane force can be viewed as a result of the world-volume gauge coupling renormalization by the open string loops.

Keywords: D-branes, Supergravity Models.

http://jhep.sissa.it/archive/papers/jhep012004035/jhep012004035.pdf

JHEP01(2004)035

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1. Introduction

Some important issues in $d = 4$ $N = 1$ supergravity with constant Fayet–Iliopoulos (FI) term are currently clarified in [1] on the basis of the superconformal approach to supergravity [2, 3, 4]. It is natural therefore to look for some extended objects in this theory that have a fraction of unbroken supersymmetry. A candidate for such a supersymmetric solution is a BPS cosmic string in the critical Einstein-Higgs-abelian gauge field model [5]. Such BPS solutions have been identified in [6] long time ago: they are known to saturate the Bogomol'nyi bound. In the presence of a single charged scalar field, the vacuum manifold has non-trivial homotopy $\pi_1 = Z$, and the theory possesses topologically non-trivial string configurations that carry U(1)-magnetic flux.

However, so far these solutions have not been embedded into supergravity and therefore it was not clear what kind of unbroken supersymmetry in what kind of supergravity explains the saturation of the bound for the cosmic strings. We will show here that cosmic strings have 1/2 of unbroken supersymmetry when embedded into an $N = 1$, $d = 4$ supergravity with constant FI terms[1] in a model that has a D-term potential for the charged Higgs field.

[1] In [7] a class of string solutions of supergravity with constant FI term was found, with 1/2 of unbroken supersymmetry. Their model describes axially symmetric solutions without the charged Higgs field ϕ. Therefore these solutions are not localized near the core of the string on the scale of the inverse mass of the vector field $m \sim g\langle\phi\rangle$, as in the usual local cosmic strings that we are going to describe in our paper.

Such Higgs field acquires a vev away from the core of the string, which compensates the FI term. This provides a D-flatness condition, $D = g\xi - g\phi^*\phi = 0$, away from the core, for the cosmic string in the Einstein-Higgs-abelian gauge field model [5]. This solution away from the core was studied in (2+1)-dimensional supergravity in [8, 9], where it was also established that the configuration has 1/2 of unbroken supersymmetry.[2] However, so far these solutions have not been embedded into 3+1 supergravity and therefore it was not clear what kind of unbroken supersymmetry in what kind of supergravity explains the saturation of the bound for the cosmic strings.

We argue that in $N = 1$, $d = 4$ supergravity, strings produced by Higgs fields that minimize F-terms (F-term strings) cannot be BPS saturated. Hence, the only BPS saturated strings in $N = 1$, $d = 4$ supergravity are D-term strings.

On the other hand, it is known that string theories admit various BPS-saturated string-like objects in the effective $4d$ theory. These are D_{1+q}-branes wrapped on some q-cycle. We shall refer to these objects as effective D_1-strings, or D-strings for short. Then an interesting question arises. How are the string theory D-strings seen from the point of view of $4d$ supergravity? If D-strings admit a low-energy description in terms of supergravity solitons, then the only candidates for such solitons are D-terms strings, since, as we shall show, they are the only BPS saturated strings in $4d$ theory. Thus, we conjecture that the string theory D-strings (that is, wrapped D_{1+q}-branes) are seen as D-terms strings in $4d$ supergravity. We shall give various consistency checks on this conjecture, and show that it passes all the tests.

Other than serve as an intriguing connection between D-branes and supergravity solitons, our conjecture sheds new light on the properties of some non-BPS string theory backgrounds, such as, for instance, brane-anti-brane backgrounds that break all the supersymmetries. It has been suggested [10] that a BPS D_{1+q} brane can be viewed as a tachyonic vortex formed in the annihilation of a non-BPS $\mathrm{D}_{3+q} - \bar{\mathrm{D}}_{3+q}$ pair. According to this picture, the annihilation can be described as condensation of a complex tachyon, which is a state of an open string (connecting brane and anti-brane). The tachyon compensates the energy of the original brane-anti-brane system, and the tachyonic vacuum is the closed string vacuum with no branes and no open strings. The tachyon condensate Higgses the world-volume U(1) group, and there are topologically non-trivial vortex solutions that are identified with stable BPS D_{1+q}-branes. Our conjecture puts the above picture in the following light. Since according to our conjecture D_{1+q} branes are D-term strings, it immediately follows that the energy of the $\mathrm{D}_{3+q} - \bar{\mathrm{D}}_{3+q}$-system must be seen from the point of view of the $4d$ supergravity as D-term energy, with FI D-term determined by D_{3+q} brane tension. Existence of a non-zero FI term implies that the U(1)-symmetry Higgsed by the tachyon is seen on the $4d$ supergravity side as gauged R-symmetry. We will raise here the issues related to the gauged R-symmetry in supergravity with constant FI term.

In short, some parts of our correspondence can be summarized as follows:

Wrapped D_{1+q}-branes	$\leftrightarrow$	D-term strings
Energy of $\mathrm{D}_{3+q} - \bar{\mathrm{D}}_{3+q}$-system	$\leftrightarrow$	FI D-term

[2]We are grateful to J. Edelstein for informing us about these papers.

Open string tachyon	$\leftrightarrow$	FI-canceling Higgs
U(1)-symmetry Higgsed by tachyon	$\leftrightarrow$	gauged R-symmetry
Ramond-Ramond-charge of D-string	$\leftrightarrow$	topological axion charge of D-term string.

Finally, we suggest that the proposed picture may give useful information about the form of the tachyon superpotential. The D-term nature of brane-anti-brane energy indicates that the tachyon must have a partner, a chiral superfield of opposite U(1)-charge. By mixing with this superfield in the superpotential, the tachyon gets a positive mass^2 term when branes are far apart. In this picture, the inter-brane interaction potential should be interpreted as the renormalization of D-term energy (due to renormalization of the world-volume gauge coupling) via the one-loop open string diagram.

2. Unbroken supersymmetry in supergravity and BPS equations

The supergravity model is defined by one scalar field ϕ, charged under U(1), with $K = \phi^*\phi$ and superpotential $W = 0$, so that we reproduce the supergravity version of the cosmic string in the critical Einstein-Higgs-abelian gauge field model [5]. This model can be also viewed as a D-term inflation model [11] in which the second charged field is vanishing everywhere at the inflationary stage as well as at the exit stage, which leads to a vanishing superpotential. In such case, the bosonic part of the supergravity action is reduced to

$$e^{-1}\mathcal{L}_{\rm bos} = -\frac{1}{2}M_P^2 R - \hat\partial_\mu\phi\,\hat\partial^\mu\phi^* - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V^D\,, \tag{2.1}$$

where the D-term potential is defined by

$$V^D = \frac{1}{2}D^2 \qquad D = g\xi - g\phi^*\,\phi\,. \tag{2.2}$$

Here W_μ is an abelian gauge field,

$$F_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu\,, \qquad \hat\partial_\mu\phi \equiv (\partial_\mu - \mathrm{i}gW_\mu)\phi\,. \tag{2.3}$$

The transformation rules of the fermions[3] in a model with the vanishing superpotential and trivial kinetic function for the vector multiplets are

$$\begin{aligned}
\delta\psi_{\mu L} &= \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}(e)\gamma_{ab} + \frac{1}{2}\mathrm{i}A_\mu^B\right)\epsilon_L\,,\\
\delta\chi_L &= \frac{1}{2}(\not\partial - \mathrm{i}g\not W)\phi\epsilon_R\,,\\
\delta\lambda &= \frac{1}{4}\gamma^{\mu\nu}F_{\mu\nu}\epsilon + \frac{1}{2}\mathrm{i}\gamma_5 D\epsilon\,.
\end{aligned} \tag{2.4}$$

Here the gravitino U(1) connection A_μ^B plays an important role in the gravitino transformations.

$$\begin{aligned}
A_\mu^B &= \frac{1}{2M_P^2}\mathrm{i}\left[\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi\right] + \frac{1}{M_P^2}W_\mu D\\
&= \frac{1}{2M_P^2}\mathrm{i}\left[\phi\hat\partial_\mu\phi^* - \phi^*\hat\partial_\mu\phi\right] + \frac{g}{M_P^2}W_\mu\xi\,.
\end{aligned} \tag{2.5}$$

[3]The detailed supersymmetry transformation for the general class of models will be presented in [1].

We are looking for the configuration for which all variations of fermions in (2.4) are vanishing for some non-vanishing ϵ, i.e. $\delta\psi_\mu = 0$, $\delta\chi = 0$ and $\delta\lambda = 0$. We choose the following bosonic configuration:

$$\phi(r,\theta) = f(r)\,\mathrm{e}^{\mathrm{i}n\theta}\,, \tag{2.6}$$

where θ is an azimuthal angle, and $f(r)$ is a real function that outside the string core approaches the vacuum value $f^2 = \xi$ for which the D-term vanishes. The plane of the string is parametrized by r and θ, where $x = r\cos\theta$ and $y = r\sin\theta$. We consider a metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}z^2 + \mathrm{d}r^2 + C^2(r)\mathrm{d}\theta^2\,. \tag{2.7}$$

We use the vielbein with $e^1 = \mathrm{d}r$ and $e^2 = C(r)\mathrm{d}\theta$, which leads to

$$\omega_r{}^{12} = 0\,, \qquad \omega_\theta{}^{12} = -C'(r)\,. \tag{2.8}$$

For the gauge potential we take

$$gW_\mu\,\mathrm{d}x^\mu = n\alpha(r)\,\mathrm{d}\theta \quad\to\quad F = \frac{1}{2}F_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \frac{n\alpha'(r)}{g}\mathrm{d}r\,\mathrm{d}\theta = \frac{n\alpha'(r)}{gC(r)}e^1e^2\,. \tag{2.9}$$

To solve the equations for the Killing spinor we will use the following projector

$$\gamma^{12}\epsilon \;=\; \mp\mathrm{i}\gamma_5\epsilon\,. \tag{2.10}$$

The BPS equations that are the vanishing of the transformations rules (2.4) for the chiral field χ and for the gaugino λ are in r,θ coordinates

$$\begin{aligned}\Big(C(r)\partial_r \pm \mathrm{i}(\partial_\theta - \mathrm{i}gW_\theta)\Big)\phi &= 0\,,\\ F_{12} \mp D &= 0\,,\end{aligned} \tag{2.11}$$

which leads to

$$\begin{aligned}C(r)f'(r) &= \pm nf(r)\,[1-\alpha(r)]\,,\\ \frac{\alpha'(r)}{gC(r)} &= \pm\frac{g}{n}\left[\xi - f^2(r)\right].\end{aligned} \tag{2.12}$$

The gravitino BPS equation is now

$$\partial_r\epsilon = 0\,, \qquad \left[\partial_\theta - \frac{1}{2}C'(r)\gamma_{12} + \frac{1}{2}\mathrm{i}A^B_\theta\right]\epsilon_L(\theta) = 0\,, \tag{2.13}$$

where

$$A^B_r = 0\,, \qquad M_P^2A^B_\theta = nf^2(r) + \frac{n}{g}\alpha(r)D = n\xi\alpha(r) + n\,f^2\,[1-\alpha(r)]\,. \tag{2.14}$$

Using (2.12), the value of A^B_θ is

$$A^B_\theta = \frac{n}{M_P^2}\left[\xi\alpha(r) - \frac{C(r)}{2g^2}\left(\frac{\alpha'}{C(r)}\right)'\right]. \tag{2.15}$$

A globally well-behaving spinor parameter is

$$\epsilon_L(\theta) = \mathrm{e}^{\mp\frac{1}{2}\mathrm{i}\theta}\epsilon_{0L}\,, \tag{2.16}$$

where ϵ_{0L} is a constant satisfying the projection equation (2.10). Thus to solve the gravitino equation $\delta\psi = 0$ we have to request that

$$1 - C'(r) = \pm A^B_\theta\,. \tag{2.17}$$

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3. Limiting cases

In the limiting case where $f(r)$ is a constant, we also find that $\alpha(r)$ is constant and the BPS equations give the values

$$f^2 = \xi\,, \qquad D = 0\,, \qquad \alpha = 1\,, \qquad A^B_\theta = \frac{n\xi}{M_P^2}\,, \qquad C(r) = r\left(1 \mp \frac{n\xi}{M_P^2}\right). \tag{3.1}$$

This is a cosmic string solution of the Einstein-Higgs-abelian gauge field model [5] far away from the core of the string at large r.

$$\phi = \sqrt{\xi}\mathrm{e}^{\mathrm{i}n\theta}\,, \qquad W = \frac{n}{g}\,\mathrm{d}\theta\,, \qquad F = 0\,. \tag{3.2}$$

This solution shows that asymptotically the string creates a locally-flat conical metric with an angular deficit angle[4] which is due to the constant FI term ξ:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}z^2 + \mathrm{d}r^2 + r^2\left(1 \mp \frac{n\xi}{M_P^2}\right)^2 \mathrm{d}\theta^2\,. \tag{3.3}$$

Notice also that the limit $r \to \infty$ the full supersymmetry is restored because $F_{\mu\nu} = 0$, $D = 0$, $\partial_r\phi = \hat{\partial}_\theta\phi = 0$ and and $R_{\mu\nu}{}^{ab} = 0$ which corresponds to the enhancement of supersymmetry away from the core of the string.

The other limiting case describing the string at small r is

$$f = 0\,, \qquad D = g\xi\,, \qquad \alpha' = \pm\frac{g^2\xi}{n}C(r)\,, \qquad 1 - C'(r) \mp nM_P^{-2}\xi\alpha(r) = 0\,, \tag{3.4}$$

which can be solved with two integration constants a and b as

$$\begin{aligned} C(r) &= \frac{M_P}{\xi g}\left[a\sin\left(g\xi M_P^{-1}r\right) + b\cos\left(g\xi M_P^{-1}r\right)\right]\,, \\ \alpha(r) &= \pm\frac{M_P^2}{n\xi}\left[1 - a\cos\left(g\xi M_P^{-1}r\right) + b\sin\left(g\xi M_P^{-1}r\right)\right]\,. \end{aligned} \tag{3.5}$$

For the vortex centre, it is required that at $r = 0$, we have $\alpha(0) = C(0) = 0$. This gives that $a = 1$ and $b = 0$, and the solution is thus:

$$\begin{aligned} \alpha(r) &= \pm\frac{M_P^2}{n\xi}\left(1 - \cos\left(\frac{g\xi}{M_P}r\right)\right)\,, \\ C(r) &= \frac{M_P}{\xi g}\sin\left(\frac{g\xi}{M_P}r\right)\,. \end{aligned} \tag{3.6}$$

Notice that we have also $C'(0) = 1$, and the metric at small r is non-singular:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}z^2 + \mathrm{d}r^2 + \left[\frac{M_P}{\xi g}\sin\left(\frac{g\xi}{M_P}r\right)\right]^2 \mathrm{d}\theta^2\,. \tag{3.7}$$

[4] From unbroken supersymmetry alone we would have $C = r\left(1 \mp \frac{n\xi}{M_P^2}\right)$. It will be explained in the next section why the sign choice $\pm$ is equal to the sign of n, and therefore only the choice $-n$ for positive n and $+n$ for negative n are permitted by the positivity of energy.

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At small r the solution is

$$\phi = 0\,, \qquad W_\theta = \pm\frac{M_P^2}{g\xi}\left(1-\cos\left(\frac{g\xi}{M_P}r\right)\right)\,, \qquad F_{r\theta} = \pm M_P \sin\left(\frac{g\xi}{M_P}r\right)\,. \tag{3.8}$$

In this limit the spacetime curvature is not vanishing, $R = 2C''/C = -2\,(g\xi/M_P)^2$ and half of the supersymmetry is broken as one can see, e. g. from the integrability condition for the existence of the Killing spinor.

4. Energy and BPS conditions

The energy of the string is (where the sums over μ,ν run only over r,θ only)

$$\begin{aligned}\mu_{\rm string} = & \int \sqrt{\det g}\;{\rm d}r{\rm d}\theta\left[(\hat\partial_\mu\phi^*)(\hat\partial^\mu\phi)+\frac{1}{4}F_{\mu\nu}\,F^{\mu\nu}+\frac{1}{2}D^2+\frac{M_P^2}{2}R\right]+\\ & + M_P^2\left(\int {\rm d}\theta\sqrt{\det h}\;K\Big|_{r=\infty} - \int {\rm d}\theta\sqrt{\det h}\;K\Big|_{r=0}\right)\,,\end{aligned} \tag{4.1}$$

where K is the gaussian curvature at the boundaries (on which the metric is h). These boundaries are at $r=\infty$ and $r=0$. For the metric (2.7):

$$\sqrt{\det g} = C(r)\,, \qquad \sqrt{\det g}\,R = 2C''\,, \qquad \sqrt{\det h}\,K = -C'\,. \tag{4.2}$$

Eq. (4.1) can be rewritten by using the Bogomol'nyi method as follows

$$\begin{aligned}\mu_{\rm string} = & \int {\rm d}r{\rm d}\theta\, C(r)\left\{|(\hat\partial_r\phi \pm {\rm i}C^{-1}\,\hat\partial_\theta)\phi|^2 + \frac{1}{2}\left[F_{12}\mp D\right]^2\right\}+ \\ & + M_P^2\int {\rm d}r{\rm d}\theta\,\left[\partial_r\left(C'\pm A_\theta\right)^B \mp \partial_\theta A_r^B\right] - M_P^2\int {\rm d}\theta\, C'\Big|_{r=\infty} + M_P^2\int {\rm d}\theta\, C'\Big|_{r=0}\,,\end{aligned} \tag{4.3}$$

where we have used the explicit form of the metric (2.7). The first line vanishes by the BPS equations (2.11). The gravitino BPS equation (2.17) implies that the first term in the second line in (4.3) vanishes. The energy is thus given by the difference between the boundary terms at $r=0$ and at $r=\infty$. Using eqs. (3.3) and (3.7) we find

$$\mu_{\rm string} = 2\pi M_P^2\left(C'\big|_{r=0} - C'\big|_{r=\infty}\right) = \pm 2\pi n\xi\,. \tag{4.4}$$

To have the energy positive, $\mu_{\rm string} > 0$, we have to require, in addition to unbroken supersymmetry, that the metric has to be taken in the form $C = r(1-|n|\xi/M_P^2)$. From unbroken supersymmetry alone we would have $C = r(1\mp n\xi/M_P^2)$. From consideration of the positivity of energy we deduce that the deficit angle has to be positive and therefore only the choices $-n$ for positive n and $+n$ for negative n in $C(r)$ are possible.

This situation with positive deficit angle has an exact analogy with supersymmetric charged black holes: the mass of the black hole M can be positive or negative, while the condition of unbroken supersymmetry, $\delta\psi = 0$ can be satisfied. However, the ADM mass of the black hole has to be positive and therefore only the case of $M = |Z| > 0$, where $|Z|$ is the absolute value of the central charge, is physical. The same happens for cosmic strings: only positive deficit angle means positive energy of the configuration, $\mu_{\rm string} = 2\pi|n|\xi$.

The energy of the string, which may also defined as in [5]

$$\mu_{\text{string}} = \int \mathrm{d}r\mathrm{d}\theta\sqrt{\det g}\,T_0{}^0 = 2\pi|n|\xi\,, \tag{4.5}$$

where [6]

$$T_0{}^0 = \left\{|(\hat{\partial}_r\phi \,\pm\, \mathrm{i}C^{-1}\,\hat{\partial}_\theta)\phi|^2 + \frac{1}{2}\left[F_{12}\,\mp D\right]^2\right\} \pm M_P^2\left[\partial_r A_\theta^B - \partial_\theta A_r^B\right], \tag{4.6}$$

Note that if we use for the definition of the energy only the energy of the non-gravitational fields ϕ and W_μ, the role of the gravitino supersymmetry transformation remains obscure. The contribution to the energy comes from the surface term $M_P^2\left[\partial_r A_\theta^B - \partial_\theta A_r^B\right]$ and is due to the vector field, which at large r is defined by $A_\theta^B = \frac{1}{2M_P^2}\mathrm{i}\left[\phi\hat{\partial}_\theta\phi^* - \phi^*\hat{\partial}_\theta\phi\right] + \frac{g}{M_P^2}W_\theta\xi = \frac{n\xi}{M_P^2}$. The first two terms in (4.6) vanish due to saturation of the BPS bounds, $(\hat{\partial}_r\phi \,\pm\, \mathrm{i}C^{-1}\,\hat{\partial}_\theta)\phi = 0$ and $F_{12} \mp D = 0$. These two bounds are saturated since the supersymmetry variation of the chiral fermion χ and gaugino λ has to vanish.

The definition of the energy of the string that we are using in (4.1), which is valid for time independent configurations, is

$$E = \int_M \sqrt{\det g}\left(\frac{M_P^2}{2}R - L_{\text{matter}}\right) + M_P^2\int_{\partial M}\sqrt{\det h}\,K\,. \tag{4.7}$$

Now we see that the term $(\frac{M_P^2}{2}R - L_{\text{matter}})$ produced in addition to two BPS bounds in (4.6) also a term $[\partial_r(C' \pm A_\theta)^B \mp \partial_\theta A_r^B]$. Due to the gravitino BPS bound, (2.17), the surface term $\partial_r A_\theta^B$ in $T_0{}^0$ is cancelled by the Einstein term $\sqrt{g}\,R$. This is not surprising since the Einstein equation of motion must be satisfied due to vanishing gravitino transformations. The remaining term in the energy, the Gibbons-Hawking K surface term, gives the non-vanishing contribution to the energy of the string which is directly related to the deficit angle Δ, where $M_P^2\Delta = \mu_{\text{string}}$.

5. Are D-term strings the D-strings?

Note that in $N = 1$ supersymmetric theories, one could easily come up with non-trivial superpotentials such that the minimization of F-term(s) triggers the spontaneous breaking of some gauge U(1)-symmetries. Such theories would also admit topologically non-trivial string solutions, which we shall refer to as the F-term strings. In the core of the F-term strings some of the F-terms become non-zero. It is important to note that F-term strings, as opposed to D-term strings *cannot* be BPS saturated objects, in the sense that they break all the supersymmetries. This fact can be directly understood from the gaugino transformation on the string background. Indeed, the only way to compensate the magnetic contribution to gaugino variation, is through the non-zero D-term part. This is impossible if the potential energy contribution to the string mass comes from F-terms. Let us briefly demonstrate the above statement on a simple example. We shall take a globally supersymmetric theory, with U(1) gauge group spontaneously broken by two chiral superfields ϕ_+, ϕ_- with opposite U(1) charges. The relevant superpotential is:

$$W = \lambda X(\,\phi_+\phi_- - \eta^2)\,, \tag{5.1}$$

where, λ and η^2 are positive constants and X is a U(1)-neutral chiral superfield. Generalization of this example to the supergravity case is straightforward and adds nothing to our conclusion. The potential energy of this system is

$$\begin{aligned} V &= V_F + V_D\,, \\ V_F &= \lambda^2|\phi_+\phi_- - \eta^2|^2 + \lambda^2|X|^2|\phi_-|^2 + \lambda^2|X|^2|\phi_+|^2\,. \\ V_D &= \frac{g^2}{2}\left[|\phi_+|^2 - |\phi_-|^2\right]^2, \end{aligned} \tag{5.2}$$

which is minimized for $\phi_+ = \phi_- = \eta$ and $X = 0$, and with unbroken supersymmetry and spontaneously broken U(1) gauge group. Hence there are topologically stable strings in this model. Because of the opposite charges, ϕ_+ and ϕ_- will have opposite winding of the phase around the string, that is the string solutions will have a form $\phi_\pm = f_\pm(r)\mathrm{e}^{\pm\mathrm{i}\theta}$. Let us study the energy and the supersymmetric properties of these objects. First, we shall note that in the lowest energy configuration $|X|$ must vanish, as it only multiplies positive definite terms in the potential. Hence, we can simply set $X = 0$.

Next, we note that because the system is invariant under the exchange $\phi_+ \to \phi_-^*$, and because the D-term is minimized for $|\phi_-| = |\phi_+|$, the lowest possible energy string configuration must have $f_+ = f_-$. Hence, we can set $\phi_+ = \phi_-^*$. The non-zero part of the string energy can only satisfy the Bogomol'nyi limit provided we set $\lambda^2 = g^2/2$. With this choice of the parameters and with the above ansatz, the energy of the scalar fields becomes identical to the one given by (2.1), where the role of ξ is played by $2\eta^2$. We have shown above that such a system does have the string solution that energetically satisfies the BPS bound.

Although it formally satisfies the energy bound, nevertheless, such a configuration breaks all the supersymmetries. To see this, it is enough to check the transformation properties of the gaugino, and the chiral fermionic partner of X (we shall call it χ) in the string background:

$$\begin{aligned} \delta\chi_L &= -\frac{1}{2}F_X\epsilon_L\,, \\ \delta\lambda^\alpha &= \frac{1}{4}\gamma^{\mu\nu}F_{\mu\nu}\epsilon\,. \end{aligned} \tag{5.3}$$

Here $F_X = \lambda(\phi_+\phi_- - \eta^2)$ is the F-term of X. Because, both $F_{\mu\nu}$ as well as F_X are non-zero in the string core, (with $F_{\mu\nu}$ of magnetic type, $\gamma^{\mu\nu}F_{\mu\nu}$ is invertible) the above variations cannot vanish for any choice of ϵ, and hence F-term strings break all the supersymmetries. Even if the parameters are fine tuned in such a way that at the tree level such strings satisfy the Bogomol'nyi energetic bound, nevertheless they are not BPS saturated objects in the sense of unbroken supersymmetry. Since all the supersymmetries are broken, the tree level Bogomol'nyi bound is not protected against quantum corrections and in general will be destabilized in perturbation theory, since the physical gauge and scalar couplings are renormalized differently. Hence, the choice $\lambda^2 = g^2/2$ is not perturbatively stable.

In supergravity the story is very similar. Let us choose a minimal Kähler $K = |\phi_+|^2 + |\phi_-|^2 + |X|^2$. For the F-term strings, in order to achieve the saturation of the Bogomol'nyi energy bound, even in the case of a minimal Kähler, we have to allow for a non-minimal gauge kinetic function, for reasons that will become clear in a moment. Again, just as in rigid limit, it can be shown easily that making either $|X| \neq 0$, or $\phi_+ \neq \phi_-^*$ only increases the energy. Thus, we set $X = 0$, and $\phi_+ = \phi_-^* = \phi/\sqrt{2}$, and the static z-independent energy of the system becomes

$$E = \hat{\partial}_\mu \phi \, \hat{\partial}^\mu \phi^* + \frac{\operatorname{Re} f(\phi)}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\lambda^2}{4} \mathrm{e}^{|\phi|^2 M_P^{-2}} \left(|\phi|^2 - 2\eta^2\right)^2 , \tag{5.4}$$

where $\mu, \nu = 1, 2$. Again this expression can be simply reduced to the energy of a single scalar field, with a FI D-term potential with $\xi = 2\eta^2$, provided the gauge kinetic function is chosen to be exactly equal to

$$\operatorname{Re} f(\phi)^{-1} = \frac{\lambda^2}{2g^2} \mathrm{e}^{|\phi|^2 M_P^{-2}} . \tag{5.5}$$

This condition is analogous to the choice $\lambda^2 = 2g^2$ in the rigid limit. Such a system admits the string solutions that satisfy Bogomol'nyi equations. In the $\xi \ll M_P^2$ limit, the solution in leading order coincides with our D-term cosmic string solution up to corrections of order ξM_P^{-2}. Again, although the solution satisfies an energetic BPS bound, it is *not* a BPS state in the sense of an unbroken supersymmetry, since it breaks all the supersymmetries.

To summarize, by a special choice of the Kähler and gauge kinetic function, the F-term string energy can formally made equal to some D-string energy, and thus, saturate the Bogomol'nyi energy limit. However, the supersymmetric properties of the two solutions are very different, due to the fact that the participating fields belong to the different multiplets. F-term strings break all the supersymmetries, whereas the D-term strings preserve half of it. This is also reflected in the fact that the choice (5.5) is not stable under radiative corrections, and hence the bound is not expected to be maintained to all orders in perturbation theory, whereas for the D-term strings, the equality of the gauge coupling to the Higgs self-coupling is imposed by supersymmetry.

It is obvious that the addition of an FI term cannot change this situation, as long as $\lambda \neq 0$. First, because the exact cancellation between the D-term and $F_{\mu\nu}$ in the gaugino variation will be impossible. Secondly, for $\lambda \neq 0$, F_X cannot be made identically zero throughout the solution.

Another argument showing that F-term strings cannot be BPS-saturated in supergravity theories, is based on the gravitino transformation properties in (2.4). The crucial point, that enabled the existence of Killing spinors on the space with a conical deficit angle, is the conspiracy between gauge and spin connections. This was possible only because the gravitino change under the broken U(1) was set by FI term, which in the same time sets the string tension, and thus, the deficit angle. Hence, transporting the spinor around the string, the phases acquired due to the U(1) charge and due to the deficit angle cancel, and the existence of the Killing spinors is possible [8]. On the other hand if the contribution into the string tension were coming from the F-terms, this cancellation would be impossible, since the U(1)-charge of gravitino only comes from FI term, and is independent of the value of F-terms.

Having argued that D-term strings are the only BPS-saturated strings in $N = 1$, $d = 4$ theory, we wish to point out a possible intriguing connection, between the D-term strings, and D_1-strings in type IIB string theories (see [12] for a review). Namely, we wish to conjecture that the D_1-strings as seen from the point of view of the low-energy supergravity are in fact D-term strings. This connection can shed some new light on D-string formation in D-brane annihilation.

To explain the connection, let us recall first some of the known facts of unstable brane-anti-brane systems [10]. Consider for simplicity a $D_p - \bar{D}_p$ pair in type II string theory. We shall assume that 6 dimensions are compactified on a torus, and the D-branes can move relative to each other in some of them. When branes are close to each other, the open string connecting them has a tachyon in the spectrum, which signals instability. It was conjectured [10] that the annihilation can be described in terms of the tachyon condensation. The topology of the tachyonic vacuum is non-trivial and can be described in terms of K-theory [13].

For a single brane-anti-brane pair, the low-energy symmetry group consists of two U(1) symmetries, and the tachyon is charged under a linear combination of these. After tachyon condensation, the diagonal U(1) subgroup gets Higgsed. The tachyon is complex, but the gauge invariance implies that the potential energy can only depend on its absolute value. Hence the tachyonic vacuum has a non-trivial π_1 homotopy, and there must exist stable tachyonic string solutions. These strings must lie in the plane of the parent branes, and have co-dimensions two less than the dimension of these branes. They are identified with the stable BPS D_{p-2} branes. In our case, since all the dimensions are compact these objects will effectively look as D_1 strings from the four-dimensional point of view. The question is what these strings correspond to as seen from the four-dimensional supergravity. It is tempting to assume that they should correspond to some stable BPS strings in that theory. But if so, we have argued that the only possible BPS strings in supergravity are the D-term strings, and hence they are the only candidates for D-strings.

This fits the picture. Indeed, the energy density of the $D_p - \bar{D}_p$ system breaks all the supersymmetries. According to the tachyonic description, supersymmetry is restored by the tachyon condensation, which cancels the energy of the original system. The tachyonic vacuum must possess stable BPS D_{p-2} branes, which must correspond to BPS strings in effective four-dimensional supersymmetric theory. But we have shown above that the only candidate for such strings are the D-term strings. Hence the initial energy of the brane-anti-brane system compensated by the tachyon condensate must exist in form of the D-term rather than the F-term, or else tachyonic strings would be non-BPS states.

The Ramond-Ramond charges of the D_{p-2} branes that in the D-brane description come from the coupling of the $(p-1)$-form RR field ($C_{(p-1)}$) to the U(1) magnetic flux on the world volume of the unstable brane-anti-brane pair

$$\int_{p+1} F_{(2)} \wedge C_{(p-1)} \tag{5.6}$$

at the level of an effective $4d$ supergravity D-term strings, should be translated as charges with respect to the "axionic" multiplet, coupled to the magnetic flux. Coupling to such a multiplet should not undermine the BPS properties of the D-term strings. For instance the repulsion between the two parallel strings mediated by the pseudoscalar axion should be exactly compensated by the attraction of its scalar partner.

6. A consistency check: matching tensions

We wish to give a brief consistency check of our conjecture of D-term-D-string correspondence, and show that the D-term string tension has the correct scaling properties for the D-brane. Consider a D_1 string formed in the annihilation of a $\mathrm{D}_3 - \bar{\mathrm{D}}_3$ brane pair. According to the description in terms of the open string tachyon condensation, the D_1 brane is a tachyonic vortex, which carries the magnetic flux of the Higgsed diagonal U(1) subgroup of the original U(1) $\times$ U(1) symmetry group. The tachyonic vacuum, with no branes, is supersymmetric and the D_1 brane breaks half of it.

According to our conjecture, this picture translates as follows. The $\mathrm{D}_3 - \bar{\mathrm{D}}_3$ system is seen as the background with a non-zero FI D-term for the diagonal U(1). Hence, supersymmetry is broken by the presence of the D-term. Supersymmetry is restored by the condensation of the Higgs field that cancels the FI term. D_1-strings are just the BPS D-term strings discussed above. We shall now match the scaling properties of the corresponding parameters involved in the problem. For simplicity, we shall keep the six additional dimensions uncompactified. In this limit the effective $4d$ gravitational constant vanishes and $4d$ supergravity approaches the rigid limit. Matching of the $\mathrm{D}_3 - \bar{\mathrm{D}}_3$ tension to the D-term energy implies

$$2T_3 = \frac{2}{g_s(2\pi)^3\alpha'^2} = \frac{g^2}{2}\xi^2 \,, \tag{6.1}$$

where g_s is the string coupling. Matching of the D_1-brane and D-string tensions gives the following relation

$$T_1 = \frac{1}{g_s(2\pi)\alpha'} = \mu_{\mathrm{string}} = 2\pi\xi \,. \tag{6.2}$$

The two are consistent, provided $g^2 = 8\pi g_s$. This is indeed a correct scaling relation between string and gauge couplings. The additional factor of two is the result of the relative charge of our Higgs field (which we have normalized to one) with respect to the open string tachyon, to which in general it is related by a non-trivial field redefinition. The above matching can be generalized for BPS D-branes formed in annihilation of different $\mathrm{D}_p - \bar{\mathrm{D}}_p$ systems. For instance, for the D_{1+q}-branes formed in the annihilation of $\mathrm{D}_{3+q} - \bar{\mathrm{D}}_{3+q}$ branes, with q world-volume dimensions wrapped on a q-torus of radius R, we have the following matching conditions

$$\frac{2R^q}{g_s(2\pi)^3\alpha'^{\frac{q+4}{2}}} = \frac{g^2}{2}\xi^2 \,, \tag{6.3}$$

and

$$\frac{R^q}{g_s(2\pi)\alpha'^{\frac{q+2}{2}}} = \mu_{\mathrm{string}} = 2\pi\xi \,, \tag{6.4}$$

which is satisfied for $g^2 = 8\pi g_s \frac{\alpha'^{\frac{q}{2}}}{R^q}$.

The fact that the tensions match so precisely is expected, but it is also surprising. It is expected from the general scaling relation of string and gauge couplings that tensions of the gauge theory solitons should scale as the ones of D-branes. On the other hand the soliton in question is derived from the low-energy gauge theory, and one would naively expect a correction due to the infinite tower of states integrated out. Once again, supersymmetry and BPS properties seem to give us much better control of the situation.

7. RR charges

D_1-branes are charged with respect to the two-form RR field $C_{(2)}$. If our conjecture of D-term-D_1-string correspondence is correct, this long-range RR field must have a counterpart in the D-term string description. As noted above, an obvious candidate for such a long-range field is the axion (a). Indeed, the axion is dual to a two-form field, and it is not surprising that this two-form is exactly the RR two-form field of the D_1-brane. Thus, the electric RR-charge of the D_1-brane translates as the topological (winding) axionic charge of the D-term-string. We wish to analyse more closely how this relation comes about.

Let us again think about the formation of the D_{1+q}-brane in $\mathrm{D}_{3+q} - \bar{\mathrm{D}}_{3+q}$ annihilation. As before, we assume that q dimensions are wrapped on a compact cycle, so that the daughter D_{1+q}-brane effectively looks as D_1 brane from the point of view of $4d$ theory. The low energy symmetry group is $\mathrm{U}(1) \times \mathrm{U}(1)$, one linear superposition of which is Higgsed by the tachyon VEV. D_1 strings are vortices formed by the winding of the tachyon phase, which support the gauge flux of the Higgsed U(1). Notice that this Higgsed U(1) gauge field is precisely the combination of the original U(1)-s that carries a non-zero RR charge (the other combination is neutral). The corresponding gauge field strength ($F_{(2)}$) has a coupling to the $C_{(2)}$ form via the WZ terms on the world-volume of the unstable parent brane pair

$$\int_{3+1+q} \frac{1}{M_P} F_{(2)} \wedge C_{(2+q)} \,, \tag{7.1}$$

where, since we are interested in the effective $4d$ supergravity description, we have only kept the $4d$ zero mode component of the RR field. The above coupling ensures that the tachyonic vortex has a correct D_{1+q} RR charge. The unit magnetic flux flowing in the z direction, generates a long-range RR field, with the following asymptotic energy density in r-direction:

$$(\mathrm{d}C_{(2)})^2|_{r\to\infty} = \frac{T_1^2}{M_P^2} \frac{1}{4\pi^2 r^2} \,, \tag{7.2}$$

where T_1 is the effective one-brane tension given by (6.4), and $C_{(2)}$ is an effective two-form field obtained via dimensional reduction of $C_{(2+q)}$ in which q indices take values in compact dimensions.

This gives a log-divergent integrated energy per unit D-string length. To make connection with the D-term-string language, we can go into the dual description of the $C_{(2)}$-form in terms of an axion

$$\mathrm{d}C_{(2)} \to * \,\mathrm{d}a \,, \tag{7.3}$$

where star denotes a $4d$ Hodge-dual. Under this duality transformation we have to replace

$$(\mathrm{d}C_{(2)})^2 \to M_P^2(\mathrm{d}a - gQ_aW)^2\,, \tag{7.4}$$

where $Q_a = \xi/M_P^2$ is the axion charge under U(1). This charge vanishes as the compactification volume goes to infinity, and $4d$ supergravity approaches the rigid limit. The above value of the axionic charge for the D-term string, reproduces the correct RR charge of the D-string, and also has a correct scaling for the anomaly cancellation (see below).

Let us show that the axionic long-range energy of the D-term string exactly matches the RR long-range energy (7.2) of the D_1-brane. There are two phases in the problem, one is the phase Θ of the D-term-compensating field ϕ, and the other one is the axion a. Both are defined modulo 2π. The gradient energies of these two fields far away from the string core are given by the following terms

$$\int r\mathrm{d}r\mathrm{d}\theta\,\frac{1}{r^2}\left[\xi(\partial_\theta\Theta - gW_\theta)^2\; + M_P^2(\partial_\theta a - gQ_aW_\theta)^2\right], \tag{7.5}$$

where the configuration is given by (3.2).

Because the VEVs must be single valued, both phases must change around the string by an (integer)$\times 2\pi$. That is, we must have

$$\frac{1}{2\pi}\oint \mathrm{d}\theta\partial_\theta\Theta = n\,, \qquad \frac{1}{2\pi}\oint \mathrm{d}\theta\partial_\theta a = n_a\,. \tag{7.6}$$

Hence, the D-term string is characterized by two integer winding numbers $(n,\, n_a)$. Around the minimal D-term string we have $n = 1$. For $n = 1$, the energetically most favorable value of n_a is determined by the charge of axion. For $Q_a \neq 1$, there is no way to compensate the gradient energy by any integer n_a, and hence there is a long-range field around the string. For large r the energy density is

$$M_P^2\frac{(n_a\; - n\,Q_a)^2}{r^2}\,. \tag{7.7}$$

In the weak $4d$ gravity limit $Q_a \ll 1$, the lowest energy configuration with a unit flux, is characterized by the winding $n_a = 0$, that is, the axion prefers not to wind at all. Hence, the long-range energy behaves as

$$\frac{\xi^2}{M_P^2 r^2}\,, \tag{7.8}$$

which exactly matches the RR long-range energy in (7.2).

The fact that the D-term string has a long-range axionic field, is another consistency check of our conjecture that it is a D_1-string. The way in which the long-range field appears for the D-term string is rather profound, and naively one would not expect such a situation. We wish to go over this effect once again and look at it from a slightly different angle. The system exhibits two U(1) symmetries, which are the shifts of Θ and of a, and thus these fields are two Goldstone particles. One combination is the gauged U(1) symmetry, under which

$$\Theta \to \Theta + g\alpha(x)\,, \qquad a \to a + gQ_a\alpha(x) \qquad \text{i.e.} \qquad \delta_\alpha\Theta = g\,, \qquad delta_\alpha a = gQ_a = g\frac{\xi}{M_P^2}\,. \tag{7.9}$$

The combination

$$G = \xi\Theta\,\delta_\alpha\Theta + M_P^2 a\,\delta_\alpha a = g\xi\,(\Theta + a)\,, \tag{7.10}$$

is eaten up by the U(1) gauge field. Hence, this combination is removed by the Higgs effect. The remaining combination in the action is

$$g_{\text{axion}} \equiv \frac{1}{g}\,(\Theta\,\delta_\alpha a - a\,\delta_\alpha\Theta) = \frac{\xi}{M_P^2}\Theta - a \tag{7.11}$$

stays massless (apart of a possible anomaly, see below). Because both U(1)-symmetries are non-linearly realized, the topology of the vacuum manifold is $S_1 \times S_1$ and one expects the two basic types of cosmic strings: 1) gauge strings with magnetic flux and no long-range field; and 2) global strings with the long-range g_{axion}-field, but no magnetic flux. One may think that the gauge strings carry no long-range field, because the winding of G exactly compensates the vector potential, and accordingly, the global strings carry no flux because G does not wind around them. However, this is impossible. The reason is that it is Θ and a, and not G and g_{axion} that must be single valued around the string. And because, a carries a fractional charge, the former requirement does not imply the latter. Hence, the gauge strings with flux also carry a long-range field, and vice versa. This property gives another consistency check of the conjecture that D-term strings are indeed D-strings, that carry long-range RR-flux.

Now let us briefly discuss what happens with the D-term strings in case that the anomaly is canceled by the Green-Schwarz [14] mechanism. In such a case a_{axion} has a coupling to *all* the gauge field strengths that have non-zero mixed anomalies

$$g_{\text{axion}}\,F\tilde{F} = \left(\frac{\xi}{M_P^2}\Theta - a\right)F\tilde{F}\,. \tag{7.12}$$

Note that the first term is generated by the chiral anomaly which is canceled by the second one. The above coupling generates a non-trivial potential for g_{axion} through the instantons, which breaks the continuous shift symmetry down to a subgroup (defined by an anomaly-free discrete subgroup of the global U(1) symmetry), and strings become boundaries of the domain walls. Again the similar instability is known to take place for D-strings in type II string theory [15], and this is another consistency check of our conjecture.

8. R-symmetry

Our conjecture of D-term-D-string correspondence leads us to the conclusion that the U(1) symmetry Higgsed by the tachyon VEV is in fact a gauged R-symmetry. We shall now try to analyse this connection in more detail. Again, we shall take tachyon condensation in $D_{3+q} - \bar{D}_{3+q}$ system as an example, assume that the six extra dimension are compactified on a torus, and keep the compactification volume as a free parameter.

We start discussing the infinite-volume case first. As noted above, in this limit the $4d$ supergravity approaches the rigid limit ($M_P \to \infty$). The $\mathrm{D}_{3+q} - \bar{\mathrm{D}}_{3+q}$ system breaks all supersymmetries, and, according to our picture, in the world-volume theory this breaking is seen as the spontaneous breaking by the non-zero FI term ξ given by the equations (6.1) and (6.2). In this limit, obviously, the $4d$ gravitino is decoupled from the U(1)-gauge field. Let us now make the volume finite. The general expression for the covariant derivative on the gravitino is in our conventions [4]

$$\mathcal{D}_{[\mu}\psi_{\nu]} = \left(\partial_{[\mu} + \frac{1}{4}\omega_{[\mu}{}^{ab}(e)\gamma_{ab} + \frac{1}{2}\mathrm{i}A^B_{[\mu}\gamma_5\right)\psi_{\nu]}\,. \tag{8.1}$$

where the U(1)-connection A^B_μ is given by

$$A^B_\mu = \frac{1}{2}\mathrm{i}\left[(\partial_i\mathcal{K})\hat{\partial}_\mu z^i - (\partial^i\mathcal{K})\hat{\partial}_\mu z_i\right] + \frac{g\xi}{M_P^2}W_\mu\,, \tag{8.2}$$

where

$$\hat{\partial}_\mu z_i = \partial_\mu z_i - W_\mu\,\eta_i(z)\,. \tag{8.3}$$

Here, $\mathcal{K}$ is the Kähler function, and sum runs over all the chiral superfields z_i and $\eta_i(z)$ are the holomorphic functions that set the U(1) transformations of all chiral superfields in the superconformal action,

$$\delta z_i = \eta_i(z)\alpha(x)\,. \tag{8.4}$$

In case that the Kähler potential is U(1)-invariant, the U(1) gauge transformation of the gravitino gauge-connection A^B_μ takes a universal form:

$$\delta A^B_\mu = \frac{g\xi}{M_P^2}\delta W_\mu = \frac{g\xi}{M_P^2}\partial_\mu\alpha(x)\,. \tag{8.5}$$

This tells us that always when the constant FI term ξ is present, the gravitino transforms non-trivially under U(1), and hence U(1) is an R-symmetry.

In the example of the section 2, we took a minimal Kähler $\mathcal{K} = zz^*$, with a single superfield $z = \phi/M_P$, in which case (8.2) reproduces (2.5). Now we have to include an additional chiral superfield (call it Y), such that the imaginary part of its scalar component is the axion $Y = \sigma + \mathrm{i}a$. The real part of Y comes from some combination of fields in NS-NS sector, we shall discuss this issue briefly below. Because of the shift symmetry, the Kähler potential must depend on a combination $Y + Y^*$. We see that, since the axion shifts under U(1): $\eta_Y = \mathrm{i}gQ_a/2$ there is an additional volume-dependent contribution to the D-term, according to

$$D(z, z^*) = -\mathrm{i}\,M_P^2\eta^i\partial_i\mathcal{K}(z, z^*) + g\xi\,. \tag{8.6}$$

To make things more explicit, we can take for a tree-level Kähler $\mathcal{K} = \log(Y + Y^*)$ and set $\phi = 0$. Correspondingly, the U(1) connection takes the form

$$A^B_\mu = \frac{1}{2}\frac{\partial_\mu a - gQ_a W_\mu}{\sigma} + \frac{g\xi}{M_P^2}W_\mu\,. \tag{8.7}$$

Under U(1) gauge transformations the first term is invariant and the second one transforms as shown in (8.5).

In effective $N = 1$, $d = 4$ supersymmetric theory, the axionic field that is coupled to the D-term string must have a scalar partner. We shall now briefly discuss its origin on

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the type IIB string theory side. For this we can look to the pairing of the states directly in 10-dimensional type IIB sugra. The closed string states are classified by representations of the spin SO(8) group as follows. From the NS_+-NS_+ sector: a scalar, a 2-form in a 28-plet and a graviton in a 35-plet. From the R_+-R_+ sector: a scalar, an RR 2-form 28-plet, and an RR 4-form 35-plet.

This shows that the RR 4-form $C_{(4)}$, which has dimension 35, is paired up with the graviton from the NS-NS sector. Hence, the axion coupled to the D_{1+q} brane for $q = 2$ should be paired up with the combination of the volume modulus and the dilaton, since this modulus comes from the graviton multiplet in 10-dimensions. This also shows that for $q = 0$, the axion is paired up with the NS-NS $B_{(2)}$ 2-form. This is also consistent with the fact that under the D-term U(1) only C_2 shifts but not $B_{(2)}$.

It can be explicitly checked [16] that for the dimensional reduction of type IIB theory on CY, when one gets $N = 2$ theory in $d = 4$, indeed the scalar dual to $C_{\mu\nu}$ partners with the scalar dual to $B_{\mu\nu}$. Together with the dilaton/axion they form a universal hypermultiplet. A second hypermultiplet is formed by the volume modulus and its axion partner dual to $C_{\mu\nu mn}$ and scalars dual to C_{mn} and B_{mn}, for the case of $h_{(1,1)} = 1$ when C_{mn} and B_{mn} are proportional to the Kähler form (m, n denote components in extra dimensions).

When compactification is on an orientifold, the universal hyper becomes a dilaton-axion $N = 1$ superfield since $C_{\mu\nu}$ and $B_{\mu\nu}$ are projected out. In the second one, only the volume and its axion partner are left since C_{mn} and B_{mn} are projected [17].

Finally, let us briefly comment on the implications of the fact that the axion shifts under the U(1) R-symmetry. In the effective four-dimensional supergravity when the constant FI term ξ is present, R-symmetry also implies that the superpotential has an R-charge which is fixed by the gravitino charge. This leads to constraints on the superpotential. If however, there is only a field dependent D-term, as in known string theory models, such constraints should be reconsidered. These issues will be discussed in detail in [1].

9. Tachyon superpotential and inter-brane potential from the D-term

In this section, we wish to very briefly discuss some intriguing implication of our conjecture for understanding the tachyon superpotential and brane-anti-brane potential. Our discussions here will be mostly qualitative, and more details will be given elsewhere.

In our picture, the supersymmetry breaking by the non BPS brane-anti-brane system corresponds to the spontaneous supersymmetry breaking via FI D-term. We wish to ask now the following question. How our picture can account for the inter-brane potential when brane and anti-brane are far apart? Again we shall work in the decompactified limit, in which $4d$ gravity is rigid. The energy density of the system then is governed solely by the D-term energy (2.2). We wish to understand how this energy (which naively looks constant for $\phi = 0$) accounts for the inter-brane force. To answer this question, we first have to understand the low energy spectrum of the theory when branes are far apart. First, when branes are far apart, there is a (nearly massless) field corresponding to their relative motion. This mode is a combination of the lowest lying scalar modes of the open strings that are attached to a brane or anti-brane only. When the branes are far apart, the lowest lying scalar excitations of these strings are nearly massless, since they correspond to Goldstone bosons of broken translations. We are interested in the combination that corresponds to the relative radial motion of branes. In the $4d$ language this mode is a member of a chiral superfield, which we shall call X. The expectation value of X than measures the inter-brane separation (call it r) according to the following relation

$$X = M_s^2 r\,, \tag{9.1}$$

where M_s is the string scale.

Next we, of course, have two gauged U(1)-symmetries. We shall only be interested in the combination that according to our conjecture provides a non-vanishing D-term.

Let us now discuss the heavy states. Among all the heavy states, we shall only be interested in the ones whose masses depend on inter-brane separation, that is on X, in our language. The tachyon is certainly among such states. This is because the tachyon is an open string state that connects the brane and the anti-brane. The mass of this stretched open string is $M_s^2 r$. In our language, this means that the tachyon as well as other open string states get mass from the coupling to our chiral superfield X. Since according to our picture there are no non-zero F terms, the non-zero mass cannot come from the interactions in Kähler, and should come from the superpotential.

$$W = X\phi\bar{\phi}\,. \tag{9.2}$$

The U(1)-invariance and holomorphy demand that the tachyon have a partner, the chiral superfield with an opposite U(1) charge. This partner we shall call $\bar{\phi}$. This partner never gets a negative mass^2 and so has a vanishing VEV. Hence it does not affect our conclusions so far. However, the existence of the partner also provides a non-trivial consistency check. Indeed, if ϕ were the only U(1)-charged superfield in low energy spectrum, the U(1) symmetry would have a chiral anomaly, which would have to be cancelled by GS mechanism. But such a cancellation is impossible in the rigid limit, since in this limit there is no $4d$ axion! Hence, the anomaly has to absent in the rigid limit, which is precisely the case because of the existence of the partner.

Now let us turn to the inter-brane potential. The energy of the system is given by the D-term energy. This is constant at the tree-level, but not at one-loop level. At one-loop level the gauge coupling g^2 gets renormalized, because of the loops of the heavy U(1)-charged states. For instance, there are one-loop contributions from the ϕ and $\bar{\phi}$ loops. More precisely there is a renormalization of g^2 due to one-loop open string diagram, which are stretched between the brane and anti-brane. Since the mass of these strings depend on X, so does the renormalized D-term energy

$$V_D = \frac{1}{2}D^2 = \frac{g_0^2}{2}(1 + g_0^2 f(X))\xi^2\,, \tag{9.3}$$

where g_0^2 is the tree-level gauge coupling, and $f(X)$ is the renormalization function. Now remembering that X sets the inter-brane distance, and using (6.2) and the relation between string and the gauge couplings, we see that the D-term generates the D-brane potential of the form

$$V(r) = 2T_3 \left(1 + g_s f(r)\right), \tag{9.4}$$

which has a correct scaling property for the $\mathrm{D}-\bar{\mathrm{D}}$ potential, computed directly from the string theory. In particular it agrees with the fact that the brane-interaction potential can be understood as either the tree-level closed string exchange or the one-loop open string amplitude. This connection will be studied in more detail elsewhere.

10. Discussion

In conclusion, we have solved the conditions for unbroken supersymmetry in $N = 1$, $d = 4$ supergravity with the D-term potential and constant FI term. The corresponding solution is a cosmic string described in detail in [5]. We have presented the form of the energy including the gravitational energy, in which the role of all supersymmetry transformations of the fermions, including the gravitino, in saturating the BPS bound is clearly explained.

We have also argued that D-term strings are the only BPS saturated strings existing in $4d$ supergravity, and have conjectured that D-strings in type II string theory can be viewed as D-term strings from the supergravity perspective. We have provided various consistency checks of of this conjecture, and have shown that it sheds new light on some non-BPS string theory backgrounds, such as brane-anti-brane systems. The tachyonic instability of such systems can now be viewed as an instability produced by the FI D-term. Our correspondence also implies under certain conditions the existence of gauged R-symmetry on such backgrounds, which can provide a powerful constraint on the possible forms of non-perturbatively generated superpotentials. These conditions as well as a clear distinction between the situations with constant FI terms and moduli-dependent D-terms in supergravity will be presented in [1].

The study of the supersymmetric properties of the cosmic string solutions in the effective four-dimensional supergravity theory, performed in this paper, may be useful particularly, in view of the increasing interest to the cosmic string solutions in string theory, see for example [18, 19, 15].

Acknowledgments

It is a pleasure to thank P. Binétruy, G. Gabadadze, S. Kachru, A. Linde, J. Maldacena, R. Myers, J. Polchinski and A. Vilenkin for useful discussions. The work of R.K was supported by NSF grant PHY-0244728. G. D. and A.V.P. thank the Stanford Institute of Theoretical Physics for the hospitality. The research of G.D. is supported in part by a David and Lucile Packard Foundation Fellowship for Science and Engineering, and by the NSF grant PHY-0070787. The work of A.V.P. was partially supported by the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime and in part by the Federal Office for Scientific, Technical and Cultural Affairs, Belgium, through the Inter-university Attraction Pole P5/27.

Before we posted the revised version of this work, the paper by E. Halyo appeared [20], which has overlap with section 6 of our work.

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